Financial Risk Management for Social and Pension Insurance

Sample Exam Questions

General Remarks about the Exam

- The exam is a 30 min oral exam. Effective examination time will be about 25 min.
- No literature or auxiliaries whatsoever are permitted.
- Candidates will do the exam on the blackboard.
- The exam covers everything in the lecture notes, including the mathematical tools and derivations and proofs. No questions will be asked on the Lundberg Bound and Spitzers's Formula in Chapter IV, Section 5, and on p. 40 (and only p.40!) in Chapter VIII and also on the graphical evaluations in Chapter VI, pp. 16-20. Moreover, no questions will be asked on Section 4 of Chapter VIII. The rest of Chapter VIII (except p.40), and also Chapter VIIIa on empirical considerations are, however, relevant for the exam.
- The exam will usually comprise two themes in the style of the ones given hereafter.
- Each theme is composed of approx. 5 to 10 technical and non-technical questions.
- After about half of the exam time, there will be a change from the first theme to the second theme which will cover a different topic so as to provide some diversification.
- It is not necessary to know everything by heart. The ability to derive a result and explain the derivation is more important than the mere reproduction of it.
- In order to achieve top marks, it is necessary to show both good mathematical skills and a good economic understanding of the subject matter.
- Hence, non-technical questions that ask for economic interpretations are also part of the exam.
- It is strictly forbidden to the examiner to give any feedback to the candidates after the exam. So, please do not ask for it.
- Results will be published towards the end of the exam session on the myStudies platform.

Theme #1: Generic model

Let us consider the generic model, i.e.

$A_t = A_{t-1} + R_t A_{t-1} + C_t$

$L_t = L_{t-1} + \lambda_t L_{t-1} + C_t$

1. Explain the transition equation for the assets!
   ⇨ See Chapter IV, Section 2, p.19 ff.
2. Explain the transition equation for the liabilities!
   ⇨ See Chapter IV, Section 2, p.21 ff.
3. Specifically, what is the liability growth rate? Elaborate!
   ⇨ See Chapter IV, Section 3, p.27 ff.

Now, suppose that the liability consists simply of a stream of future promised cashflows, abbreviated B_t (for benefits) here: $\mathbf{B} = (B_1, \ldots, B_T)'$

4. At time zero, what is the value of these liabilities?
   ⇨ See Chapter IV, Section 3, p.28/29
5. At some time t > 0, what is the value of these liabilities?
   ⇨ See Chapter IV, Section 3, p.28/29
6. What is the liability growth rate in this setup?
   ⇨ See Chapter IV, Section 3, p.28/29
7. Can you please prove this?
   ⇨ See Chapter IV, Section 3, p.28/29
8. What happens if the estimates for B_t are biased, i.e. if the values B_t of the cashflows have to be revised over time?
   ⇨ See Chapter IV, Section 3, p.29

Theme #2: Moment-generating functions

1. How is the moment-generating function defined?

   ⇨ See Chapter V, Section 2, p.12

2. Why is this called the “moment-generating function”?

   ⇨ See Chapter V, Section 2, p.13

3. What is the ansatz (only the ansatz) to prove this?

   ⇨ See Chapter V, Section 2, p.13

4. Does the moment-generating function exist for any random variable?

   ⇨ Follows from Def. 2 in Chapter V, Section 2

5. If two random variables have the same moment-generating function, what can be said about their probability laws?

   ⇨ See Chapter V, Section 2, Proposition 3

6. What is the moment-generating function of a Normal random variable with expected value μ and variance σ^2 ?

   ⇨ See Chapter V, Section 3, p.18

7. Assume that $\mu = 0$ and $\sigma^2 = 1$ and prove this!

   ⇨ See Chapter V, Section 3, p.19

Theme #3: Probability of underfunding

We consider the lognormal model for the funding ratio with given parameters FR0 for the initial funding ratio, μ for the expected return, σ for the short-term investment risk and λ for the intrinsic growth rate of the liabilities.

1. Under the lognormal model, how is the funding ratio at time t distributed?
   ⇨ See Chapter V, Section 1, p.9/10
2. In general and independent of any model, how is the probability of underfunding defined?
   ⇨ See Chapter V, Section 4, Definition 5
3. Why is underfunding an undesirable state?
   ⇨ See e.g. Chapter IV, Section 2 or Chapter V, Section 4
4. Specifically under the lognormal model, what is the probability of underfunding?
   ⇨ See Chapter V, Section 4, Proposition 11
5. Please derive this! (If not already done...)
   ⇨ See Chapter V, Section 4, Proposition 11
6. In general, what are sensitivities?
   ⇨ See Chapter V, Section 4, p.30 ff.
7. What is the sensitivity of the probability of underfunding w.r.t. the expected return μ?
   ⇨ See Chapter V, Section 4, p.33 ff.
8. What is the sign of this sensitivity?
   ⇨ See Chapter V, Section 4, p.33 ff.
9. Can you interpret this practically?
   ⇨ See Chapter V, Section 4, p.33 ff.
10. What are the shortcomings of the probability of underfunding?
    ⇨ See e.g. Chapter V, Section 4, p.35 or Section 5

Theme #4: Funding ratio at risk

1. What is the CDF of a general lognormal random variable?
   ⇨ See Chapter V, Section 3, Proposition 7
2. In general, what is a left quantile?
   ⇨ See Chapter V, Section 5, p.39 ff.
3. What is the left quantile in the lognormal case?
   ⇨ See Chapter V, Section 5, p.39 ff.
4. Can you please prove this? (If not already done...)
   ⇨ See Chapter V, Section 5, p.39 ff.
5. What are typical values for the confidence level α used in risk management?
   ⇨ See Chapter V, Section 5, p.49 ff.
6. What about $\Phi^{-1}(\alpha)$ in this case?
   ⇨ See Chapter V, Section 5, p.49 ff.
7. In general, how is the Funding Ratio at Risk defined?
   ⇨ See Chapter V, Section 5, Definition 7
8. What is the advantage of the funding ratio at risk over the probability of underfunding?
   ⇨ See various places in Chapter V, Section 5 or Chapter VI, Section 4 and Chapter VII, Sections 4 and 5
9. What does the Funding Ratio at Risk look like in the lognormal case?
   ⇨ See Chapter V, Section 5, Proposition 16
10. As compared to the Expected Funding Shortfall, is the Funding Ratio at Risk more or less conservative, and why?
    ⇨ Follows from Chapter V, Section 5, Definitions 6 and 7 and is also illustrated by Chapter VII, Section 5.

Theme #5: Risk / return profile

1. What is the risk / return profile?
   ⇨ See Chapter VI, Section 2
2. What does the risk / return profile usually look like?
   ⇨ See Chapter VI, Section 2, in particular p.12
3. What are the formal properties of the risk / return profile?
   ⇨ See Chapter VI, Section 2, in particular p.13
4. What is the Funding Ratio at Risk in the lognormal model?
   ⇨ See Chapter V, Section 5, Proposition 16
5. Insert the risk / return profile into this!
   ⇨ See Chapter VI, Section 4, p.31 ff.
6. If we change μ, what is the overall effect on the Funding Ratio at Risk? Explain!
   ⇨ See Chapter VI, Section 4, p.31 ff.
7. Compute the overall sensitivity of the Funding Ratio at Risk w.r.t. μ, assuming we have inserted the risk / return profile?
   ⇨ See Chapter VI, Section 4, p.31 ff.
8. Under what circumstances is this negative?
   ⇨ See Chapter VI, Section 4, p.31 ff.
9. What are typical values of α, and what is the sign of $\Phi^{-1}(\alpha)$ in this case?
   ⇨ See Chapter VI, Section 4, p.31 ff.
10. If the derivative of the Funding Ratio at Risk w.r.t μ is negative (the full derivative with the risk / return profile inserted), what does this mean in practice?
    ⇨ See Chapter VI, Section 4, p.31 ff.